Reply on the Comment on "New Conditions for a Total Neutrino Conversion in a Medium"

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Chizhov and Petcov Reply: We have found in [1] *new conditions for a total neutrino conversion in a medium* (see also [2,3]). It is claimed in [4] that our results are a particular case of enhancement of neutrino oscillations, suggested in [5,6] and widely discussed in the literature. We refute these claims, confirming the novelty of our results.

We have studied in [1] the transitions $\nu_e \to \nu_{\mu(\tau)}$, etc. of neutrinos which crossed $n = 2$ (3) alternating layers with constant densities N_1 and N_2. The probability of the transitions, $P_{2(3)}(\nu_a \to \nu_b) \equiv P_{2(3)}$, is given by [1,2]:

$$P_2 = 1 - Y^2 - X_3^2, \qquad P_3 = 1 - \bar{Y}^2 - \bar{X}_3^2, \qquad (1)$$

where Y, X_3 are defined in [4] (eqs. (1), (2)), $\bar{Y} = -c_2 + 2c_1 Y$, $\bar{X}_3 = -s_2 \cos 2\theta_2 - 2 s_1 \cos(2\theta_1) Y$, c_j $(s_j) \equiv \cos\phi_j$ $(\sin\phi_j)$, θ_j and $2\phi_j$, $j = 1, 2$, being the mixing angle in matter in layer j and the neutrino state phase difference after neutrinos crossed this layer. The *new conditions for a total neutrino conversion* follow from (1) [2]:

$$n = 2: \;\; Y = 0,\; X_3 = 0; \quad n = 3: \;\; \bar{Y} = 0,\; \bar{X}_3 = 0. \qquad (2)$$

The solutions of these conditions were given in [1] (eqs. IV and (22)). We have shown also [1] that, e.g., for $n = 2$, (2) are conditions for a *maximal constructive interference* between the amplitudes of neutrino transitions in the two layers. Thus, a clear physical interpretation of the absolute maxima of P_2 is that of *constructive interference maxima*.

(i) In connection with eq. (2) and the related effect of total neutrino conversion the authors of [4] write: "... the "new effect of total neutrino conversion" [1] is nothing but a particular case of the parametric resonance enhancement of neutrino oscillations, suggested in" [5,6] "and widely discussed in the literature...". We note that the two sets of two conditions in (2) and their solutions were not derived and/or discussed in any form in [5,6] or in any other article published before [1,2]. They do not follow from the conditions of enhancement of $P(\nu_a \to \nu_b)$ found in [5,6] or [7].

(ii) For n alternating layers one has according to [4]

$$P_n(\nu_a \to \nu_b) = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + X_3^2} \sin^2 \Phi_p, \qquad (3)$$

where $\mathbf{X} = (X_1, X_2, X_3)$ is a real vector, $\mathbf{X}^2 = 1 - Y^2$; $\mathbf{X}$ and Φ_p are defined in [4] (eqs. (1) - (5)). According to [4], eq. (3) describes parametric oscillations, and parametric resonance occurs when $X_3 \equiv -(s_1 c_2 \cos 2\theta_1 + c_1 s_2 \cos 2\theta_2) = 0$, the latter being the "resonance condition". Due to eq. (1), any resonance interpretation of the probabilities $P_{2,3}$ based solely on eq. (3) seems physically questionable. Actually, the denominator in (3) is always canceled by $\sin^2 \Phi_p$, and P_n is just a polynomial without any explicit resonance features: for even n, $\sin^2 \Phi_p = \mathbf{X}^2 U^2_{n/2-1}(Y)$, $U_n(x)$ being the well-known Chebyshev polynomial of the second kind, $Y = \cos\Phi = c_1 c_2 - \cos(2\theta_2 - 2\theta_1) s_1 s_2$; for odd $n \geq 3$,

$$P_n = \left[s_1 \sin 2\theta_1 \cos\left(\frac{n-1}{2}\Phi\right) + Z U_{\frac{n-3}{2}}(Y) \right]^2, \qquad (4)$$

$Z = s_2 \sin 2\theta_2 + Y s_1 \sin 2\theta_1$. As should also be clear from (1), $X_3 = 0$ *alone does not ensure the existence even of a local maximum* of $P_{2,3}$. The same conclusion is valid for *any finite* n.

(iii) The two conditions $c_{1,2} = 0$ were not given in [5–7]; they were discussed first in [3]. What one finds in [5–7] *at most* is $2\phi_1 + 2\phi_2 = 2\pi + 2\pi k$. In addition, $c_{1,2} = 0$ are conditions of maxima of $P_{3,2}$ *only if fulfilled in a certain region of the space of parameters* [3,1,2]. Solution III is more than just $c_{1,2} = 0$ [1]: *it includes as an integral part the region* $\cos(2\theta_2 - 2\theta_1) \leq 0$*, where* $c_{1,2} = 0$ *have to hold*. Thus, solution III in [1] is not "reproducing" any of the solutions in [5–7]. Moreover [2], $c_{1,2} = 0$ *lead to a maximum of* $P_{2(3)}$ *in the neutrino energy only when they hold on the line* $\cos(2\theta_2 - 2\theta_1) = 0$ $(\cos(2\theta_2 - 4\theta_1) = 0)$. The *three* conditions represent a solution of (2) [1,2].

(iv) *There is infinite number of irrelevant solutions of* $X_3 = 0$*,* $c_{1,2} \neq 0$*, for* $n = 2, 3$. The existence of solution IV (or (22)), its explicit form, found in [1,2], could not be, and were not, inferred from the solutions of $X_3 = 0$, $c_{1,2} \neq 0$.

(v) The case of neutrino oscillations in a medium with 3 layers, studied in [3,1,2], corresponds [3] to the transitions in the Earth of the Earth-core-crossing solar and atmospheric neutrinos. Most importantly, the *new conditions for a total neutrino conversion* (2), found in [1,2], are fulfilled in this case and lead to observable effects [1–3]. The conditions of enhancement of $P(\nu_a \to \nu_b)$ obtained in [5–7] are not valid for the indicated $\nu-$transitions in the Earth.

Further comments on [4] can be found in [8].

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